Theory & Computation of Singularly Perturbed Differential Equations

IIT (BHU) Varanasi, Dec 2017

https://skumarmath.wordpress.com/gian-17/singular-perturbation-problems/

Niall Madden, NUI Galway

§0 Outline

Version 02.12.17

Outline

Monday, 4 December		
09:30 – 10.30	Registration and Inauguration	
10:45 – 11.45	1. Introduction to singularly perturbed problems	NM
12:00 – 13:00	2. Numerical methods and uniform convergence	NM
14:30 – 15:30	Tutorial (Convection diffusion problems)	NM
15:30 – 16:30	Lab 1 (Simple FEMs in MATLAB)	NM

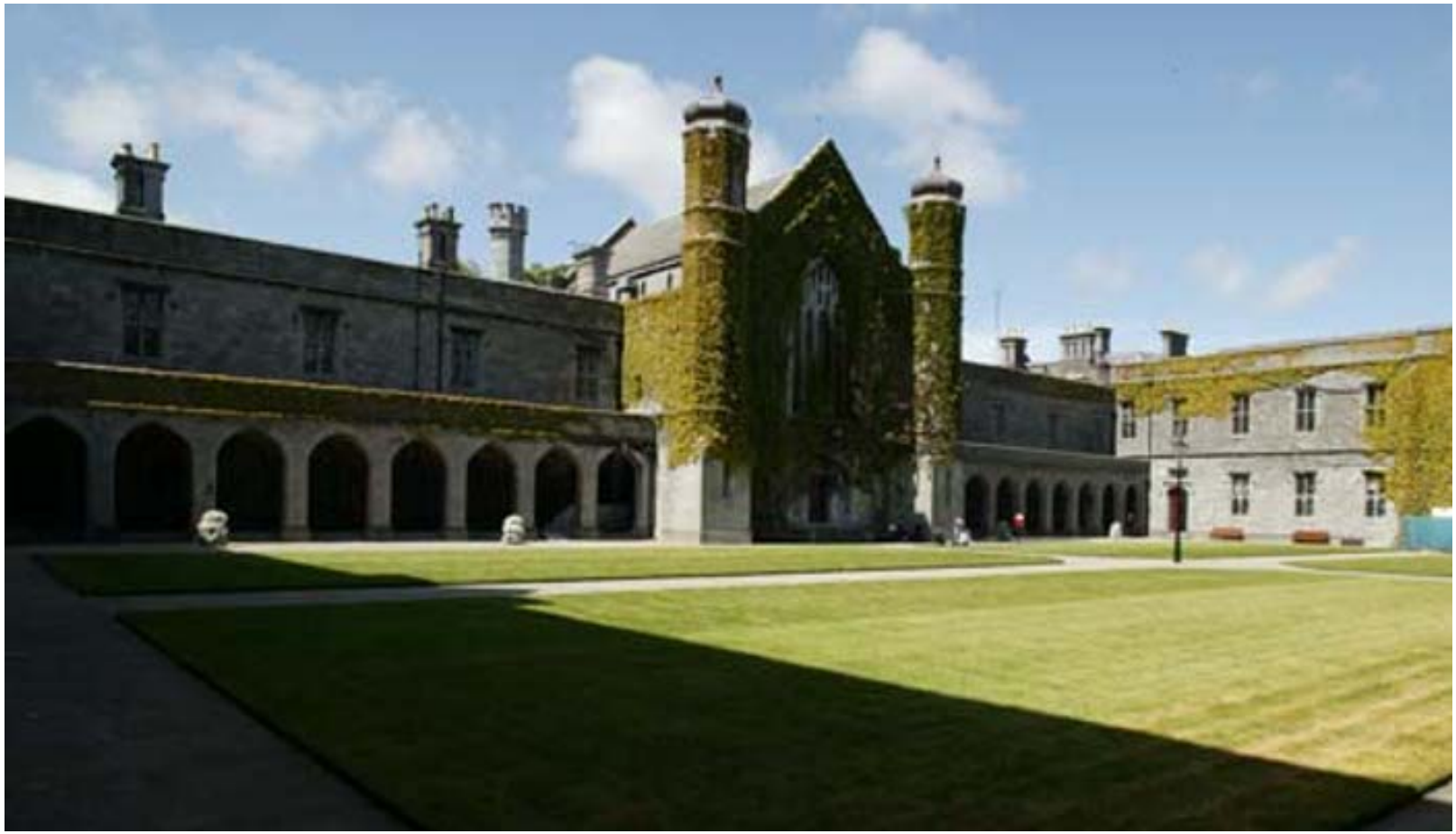
Tuesday, 5 December		
09:30 – 10:30	3. Finite difference methods and their analyses	NM
10:45 – 11:45	4. Coupled systems of SPPDEs	NM
14:00 – 16:00	Lab 2 (Fitted mesh methods for ODEs)	NM

Thursday, 7 December		
09:00 – 10:00	8. Singularly perturbed elliptic PDEs	NM
10:15 – 11:15	9. Finite Elements in two and three dimensions	NM
01:15 – 15:15	Lab 4 (Singularly perturbed PDEs)	NM

Friday, 8 December		
09:00 – 10:00	10. Preconditioning for SPPs	NM

Who am I?

My name is **Niall Madden**. I am a Senior Lecturer in Mathematics at the National University of Ireland, Galway.



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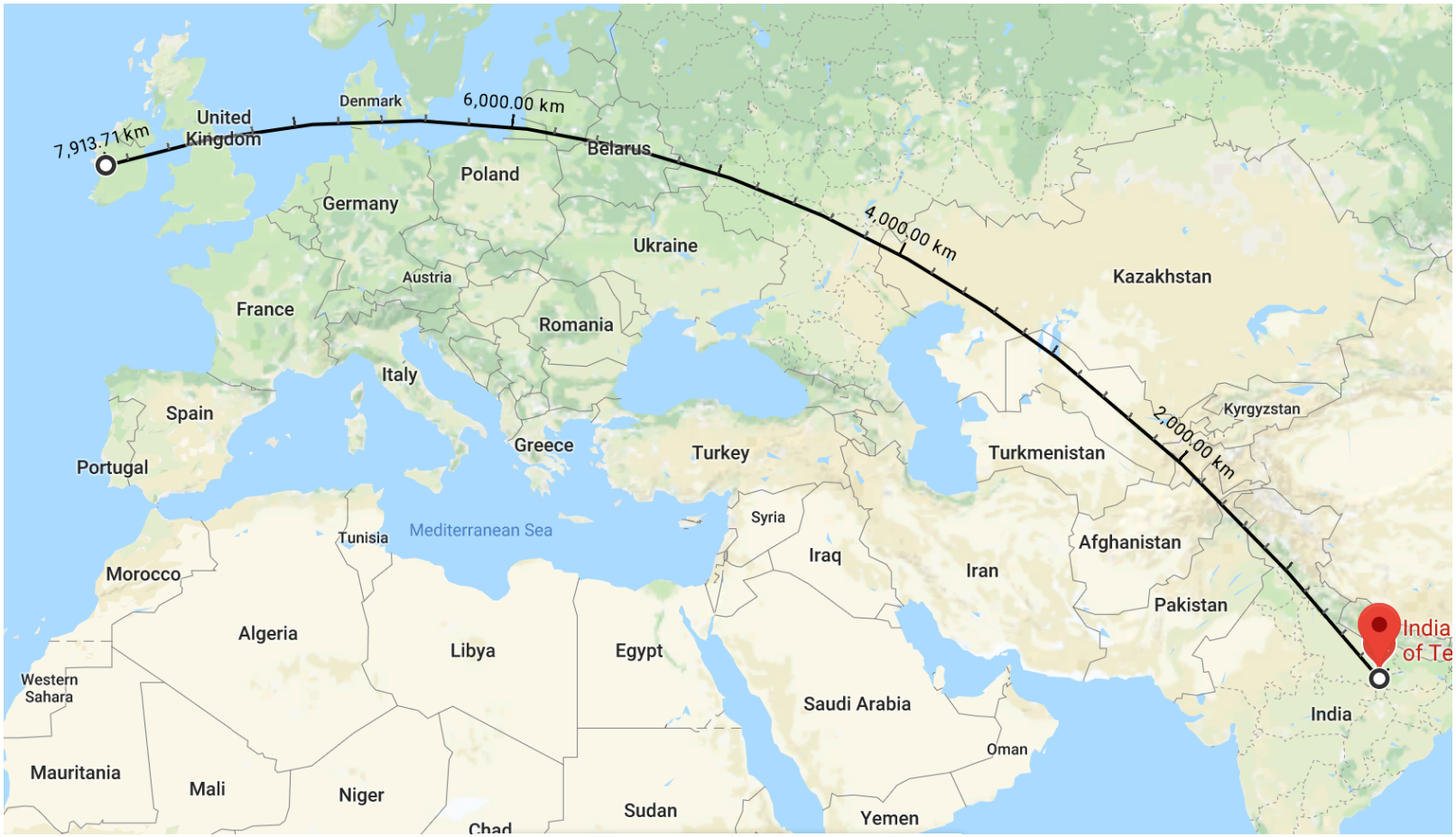

Who am I?

My name is **Niall Madden**. I am a Senior Lecturer in Mathematics at the National University of Ireland, Galway.

I completed my PhD in 2000 at University College Cork (Ireland), where I studied numerical modelling of **wave-current interactions**.

Some of the models involved singularly perturbed problems:

- 4th-order Orr-Sommerfeld Equations (hydrodynamic stability)
- k-ϵ turbulence models (expressed as coupled systems).

From this I developed an interest in the mathematics of numerical methods for these problems: particular finite difference methods for coupled systems of reaction-diffusion (ordinary differential) equations.

More recently, I have been interested in partial differential equations. Since these are "hard" (="slow") to solve, I got interested in fast methods: ADI, Domain decomposition, linear solvers, sparse grids.

Philosophy: *interesting problems come from applications; mathematical insights are key; implementation is a necessary and creative endeavour*.

Who are you?

- Mathematician? Applied Mathematician? Engineer?
- Do you prove theorems? Solve problems? Create models?
- Do you program/write code?

Credits and disclaimers

These slides were prepared by Niall Madden (Niall.Madden@NUIGalway.ie) for the **GIAN Workshop: Theory and Computation of Singularly Perturbed Problems, Varanasi, Dec 2017**.

Some of the material is original, but much of it is derived from research papers and key texts, including

- Linß: *Layer-adapted meshes for reaction-convection-diffusion problems.* 2010. [1].
- Miller, O'Riordan, Shishkin: *Fitted numerical methods for singular perturbation problems*. 2012. [2]
- O'Malley: *Think about ordinary differential equations*. 1997 [3]
- Roos, Stynes and Tobiska,: *Robust numerical methods for singularly perturbed differential equations. Convection-diffusion-reaction and flow problems.* 2008. [4]
- Shishkin and Shishkina: *difference methods for singular perturbation problems*. 2009. [5]
- Stynes: *Steady-state convection-diffusion problems*. Acta Numerica 2005. [6]

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Springer-Verlag, Berlin, 2010.

J. J. H. Miller, E. O'Riordan, and G. I. Shishkin.

Fitted numerical methods for singular perturbation problems.

World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, revised edition, 2012.

Robert E. O'Malley, Jr.

Thinking about ordinary differential equations.

Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge, 1997.

Hans-Gorg Roos, Martin Stynes, and Lutz Tobiska.

Robust Numerical Methods for Singularly Perturbed Differential Equations, volume 24 of *Springer Series in Computational Mathematics*.

Springer-Verlag, Berlin, 2nd edition, 2008.

Bibliography

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CRC Press, Boca Raton, FL, 2009.

Martin Stynes.

Steady-state convection-diffusion problems.

Acta Numer., 14:445–508, 2005.